

Low-frequency wave modulations in an electronegative dusty plasma in the presence of charge variations

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The effects of dust charge variations on low-frequency wave modulations in an electronegative dusty plasma are investigated. The dynamics of the modulated wave is governed by a nonlinear Schrödinger equation with a dissipative term. The dissipation arises due to the nonsteady (nonadiabatic) dust charge variations. Theoretical and numerical investigations predict the formation of dissipative bright (envelope) and dark solitons. The nonsteady charge-variation-induced dissipation reduces the modulational instability growth rate and introduces a characteristic time scale to observe bright solitons. Results are discussed in the context of electronegative dusty plasma experiments.

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I. INTRODUCTION

Electronegative dusty plasmas are very much common in modern technology (e.g., semiconductor material processing, development of nanomaterials, and for the efficient deposition of quality thin films) [1–6] and also are applicable in astrophysics (e.g., in the Earth's lower ionospheric plasmas and cometary plasmas) [7–9]. The dust grains produced by the chemical reaction in gas- or plasma-surface interaction in such plasmas can significantly affect the local as well as the global discharge characteristics [4,10].

In most of the astrophysical and laboratory plasmas, the electron number density decreases because of its attachment on the dust grain's surface. The presence of negative ions significantly affects the magnitudes of different plasma characteristic parameters as well as the dust charging process by modifying the surface potential acquired by a dust grain [11]. Actually, the thermal velocity of negative ions is lower than that of electrons, and as a result the presence of negative ions reduces the dust surface potential in an electronegative dusty plasma [12–14]. This causes the reduction of the magnitude of the dust charge. These modifications affect the properties of the low-frequency natural linear modes in a dusty plasma [15,16]. The nonlinear localized wave structures (e.g., a double layer) are also observed [13] in an electronegative dusty plasma.

However, in a real dusty plasma the charge (q_d) on dust grains play as an extra dynamical variable which can vary continuously with time due to continuous impact by electrons and ions (depending on the surrounding plasma environment). The instantaneous dust charge variation is known as steady-state (adiabatic) charge variation [17,18]. In this case, the dust charge instantaneously reaches its equilibrium value at each space-time point determined by the local electrostatic potential. However, the noninstantaneous charge variation causes an anomalous dissipation in a dusty plasma, which is known as nonsteady (nonadiabatic) charge variation [22]. This nonsteady charge variation causes a collisionless, non-Landau

damping of linear wave modes [15,19,20]. In the nonlinear regime, this anomalous dissipation leads to the formation of a shock wave in a dusty plasma [21–24]. In the long-wavelength limit, the shock is described by the Korteweg-de Vries Burgers equation [22,23]. The presence of negative ions modifies the nonlinear localized structures [25] and also reduces the shock strength by reducing the dust surface potential [26].

Moreover, in the short-wavelength limit, the low-frequency mode, e.g., a dust acoustic wave (DAW) [27], is modulated due to a small modulational perturbation in a dusty plasma [28–31]. A slow parallel modulation of a finite amplitude monochromatic plane wave can grow and in some limit leads to the formation of a bright (envelope) soliton. The physics behind this modulational instability is the interplay between nonlinearity and dispersion or diffraction. The nonlinearity mainly originates from the ponderomotive force. Some locally low density regions (cavity) are formed in a uniform dusty plasma due to this ponderomotive force. As a result, the DAWs are trapped within these cavities. This trapping of the plasma waves further enhances the ponderomotive force, and finally a growing envelope (bright) soliton is developed from a monochromatic plane wave in the unstable region. In the stable region, DAWs produce dark solitons or envelope holes.

In this paper, we will investigate the DAW modulations in an electronegative dusty plasma in the presence of dust charge variations. In particular, we are interested to see how the negative ions and nonsteady dust charge variations affect the DAW modulations at short wavelengths. We consider the dust fluid equations to describe the model. We also adopt the well-known reductive perturbation technique (RPT) to study the DAW modulations. The dynamics of the modulated DAW is seen to be governed by a dissipative (here damped) nonlinear Schrödinger equation (NLSE). The damping arises due to the nonsteady dust charge variations. Both analytical and numerical solutions reveal that this damping causes the soliton (bright and dark) amplitude to decay with time. The presence of negative ions reduces (increases) the magnitude of the soliton amplitude (width).

The paper is organized as follows. In Sec. II, we present the complete set of fluid equations to describe the model. The damped NLSE using RPT is derived in Sec. III. The effects

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of dust charge variations on wave modulation are discussed in Sec. IV. The soliton solutions and the effects of dust charge variations on these solutions are discussed in Sec. V. The results are briefly discussed in Sec. VI. Section VII contains the concluding part.

II. PHYSICAL ASSUMPTIONS AND BASIC EQUATIONS

We consider an unbounded, unmagnetized, and collisionless electronegative dusty plasma. The plasma constituents are the electrons, singly charged ions (positive and negative), and negatively charged dust grains of uniform size. The plasma is assumed to be in the undisturbed uniform state at $-\infty$, where electrostatic potential $\phi = 0$, electron number density $n_e = n_{e0}$, positive (negative) ion density $n_{\pm} = n_{\pm 0}$, dust density $n_d = n_{d0}$, and dust charge $q_d = -z_d e$, so that the quasineutrality condition $n_{e0} + z_d n_{d0} + n_{-0} = n_{+0}$ is satisfied.

The charge on the dust grain varies continuously with space (x) and time (t). The temperature of the dust grain is very low compared to that of the electrons (T_e) and ions ($T_{\pm}$, positive and negative). So, the dust grains are effectively cold with respect to the electrons and ions. The dust grains are moving with fluid velocity U . It is convenient to express all the variables in the nondimensional form before going to the details of the basic formalism of the problem. For this purpose, we introduce the following normalizations: $\Phi = e\phi/T_e$, $N = n_d/n_{d0}$, $N_e = n_e/n_{e0}$, $N_{\pm} = n_{\pm}/n_{\pm 0}$, $\bar{x} = x/\lambda_D$, $\bar{t} = \omega_{pd} t$, $\bar{U} = U/c_{da}$, and $\bar{q}_d = q_d/z_d e = -1 + Q_d$, Q_d is the fluctuating dust charge. Here, $\omega_{pd} (= \sqrt{z_d^2 e^2 n_{d0}/\epsilon_0 m_d})$ is the dust plasma frequency, $\lambda_D (= \sqrt{\epsilon_0 T_e/\gamma n_{+0} e^2})$ is the plasma Debye length, $c_{da} (= \sqrt{z_d T_e/\alpha_d m_d})$ is the dust acoustic speed, $\alpha_d = z_d n_{d0}/\gamma n_{+0}$, $\delta_+ = n_{e0}/n_{+0}$, $\delta_- = n_{-0}/n_{+0}$, $\sigma_{\pm} = T_{\pm}/T_e$, $\Delta = (1 - \delta_- - \delta_+)$, $\gamma = \delta_+ + 1/\sigma_+ + \delta_-/\sigma_-$, and $z = z_d e^2/4\pi\epsilon_0 r_d T_e$ is a dimensionless dusty plasma parameter (the ratio of the electrostatic energy of a dust grain of radius r_d to the electron thermal energy). Hereafter, we will be using these new variables and removing all the bars for simplicity of notations.

We are interested to study the low-frequency DAW modulations, and therefore we neglect the inertia of the electrons and ions (positive and negative) compared to the dust grains. On this slow time scale, the electrons and both the ions are in local thermodynamic equilibrium, so that their densities are modeled by the following Boltzmann distributions:

$$n_e = n_{e0} e^{\Phi}, \quad n_+ = n_{+0} e^{-\Phi/\sigma_+}, \quad n_- = n_{-0} e^{\Phi/\sigma_-}. \quad (1)$$

The normalized fluid equations for the dynamics of a DAW in a one-dimensional (1D) planar geometry in electronegative dusty plasma are

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(NU) = 0, \quad (2)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = -\frac{(Q_d - 1)}{\alpha_d} \frac{\partial \Phi}{\partial x}, \quad (3)$$

and

$$\gamma \frac{\partial^2 \Phi}{\partial x^2} = (\delta_- e^{\Phi/\sigma_-} + \delta_+ e^{\Phi} - e^{-\Phi/\sigma_+}) - \Delta N(Q_d - 1). \quad (4)$$

The other dynamical variable, the dust charge, is determined by the following normalized dust charging equation:

$$\begin{aligned} \frac{\omega_{pd}}{\nu_{ch}} \left(\frac{\partial Q_d}{\partial t} + U \frac{\partial Q_d}{\partial x} \right) &= \frac{\sigma_+ \beta_{ch}}{(1 + \sigma_+ + \mu_2^{-1})} \left[\left(1 - \frac{z Q_d}{z + \sigma_+} \right) e^{-\frac{\Phi}{\sigma_+}} \right. \\ &\quad \left. - A_+ e^{(\Phi + z Q_d)} - A_- e^{\frac{(\Phi + z Q_d)}{\sigma_-}} \right]. \end{aligned} \quad (5)$$

The charging frequency ν_{ch} is given by [26]:

$$\nu_{ch} = \frac{r_d}{\sqrt{2\pi}} \frac{\omega_{p+}^2 (z + \sigma_+)(1 + \sigma_+ + \mu_2^{-1})}{V_{t+} z \sigma_+ \beta_{ch}}, \quad (6)$$

where $\omega_{p+} (= \sqrt{n_{+0} e^2/\epsilon_0 m_+})$ is the positive ion plasma frequency, and $V_{t+} (= \sqrt{T_+/m_+})$ is the positive ion thermal velocity. The other coefficients A_+ , A_- , β_{ch} , and μ_2^{-1} are given in Appendix A [See Eqs. (A1)–(A3)].

III. NONLINEAR EVOLUTION EQUATIONS

In order to study the propagation characteristics of a nonlinear DAW in the short-wavelength limit, we introduce the following slow space and time scales [32]:

$$\xi = \epsilon(x - v_g t), \quad \tau = \epsilon^2 t, \quad (7)$$

where v_g is the group velocity of the DAW and ϵ is a small parameter that characterizes the strength of nonlinearity. This transformation separates the system into a slowly varying part associated with the amplitude of the wave and a rapidly varying part, which is the phase of the wave. The operators $\partial/\partial t$ and $\partial/\partial x$ then take the forms

$$\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \xi}$$

and

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \epsilon v_g \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau},$$

to account for the slow variations of DAW amplitude.

The dependent variables are expanded in powers of ϵ in the following way:

$$\begin{bmatrix} N \\ U \\ Q_d \\ \Phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} \begin{bmatrix} n_l^{(n)}(\xi, \tau) \\ u_l^{(n)}(\xi, \tau) \\ q_l^{(n)}(\xi, \tau) \\ \phi_l^{(n)}(\xi, \tau) \end{bmatrix} \exp[i(kx - \omega t)l]. \quad (8)$$

The reality condition for all variables is $S_{-l}^{(n)} = (S_l^{(n)})^*$, where $S_l^{(n)} = n_l^{(n)}, u_l^{(n)}, q_l^{(n)}, \phi_l^{(n)}$.

In modern technological experiments (e.g., $Ar^+ - F^-$) and also in electronegative laboratory dusty plasmas ($SiH_3^+ - SiH_3^-$) [2,3,10,15], the ratio $\omega_{pd}/\nu_{ch} \sim O(10^{-2} - 10^{-3})$, which is small. These values of ω_{pd}/ν_{ch} permit us to consider the scaling

$$\frac{\omega_{pd}}{\nu_{ch}} \sim O(\epsilon^2), \quad (9)$$

for the consistent perturbation. Using the stretching Eqs. (7) and (8) and scaling Eq. (9), we expand Eqs. (2)–(5) to obtain the n th-order reduced equations (B1)–(B4) (details are given in Appendix B). For the first order ($n = 1$ and $l = 1$), we obtain the following relations:

$$\omega n_1^{(1)} = k u_1^{(1)}, \quad (10)$$

$$\alpha_d \omega u_1^{(1)} = -k \phi_1^{(1)}, \quad (11)$$

$$\gamma(1 + k^2) \phi_1^{(1)} = \Delta(q_1^{(1)} - n_1^{(1)}), \quad (12)$$

and

$$q_1^{(1)} = -\beta_{ch} \phi_1^{(1)}. \quad (13)$$

These four relations (10)–(13) self-consistently determine the dispersion relation,

$$\omega^2 = \frac{\Delta k^2}{\alpha_d[\gamma(1 + k^2) + \Delta\beta_{ch}]}, \quad (14)$$

of the DAW in an electronegative dusty plasma. In the second order ($n = 2$ with $l = 1$), we actually get the corrections of the first-order quantities, which are written in the following matrix form:

$$\begin{aligned} [n_1^{(2)}, u_1^{(2)}, q_1^{(2)}]^T &= \left[\frac{2ik\gamma}{\Delta}, \frac{i\gamma\omega}{\Delta}, 0 \right]^T \frac{\partial \phi_1^{(1)}}{\partial \xi} \\ &- \left[\frac{k^2}{\omega^2 \alpha_d}, \frac{k}{\omega \alpha_d}, \beta_2 \right]^T \phi_1^{(2)}, \end{aligned} \quad (15)$$

where the superscript T represents the transpose of a matrix. The second order also provides the following compatibility condition:

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\omega}{k} \left[\frac{\gamma + \Delta\beta_{ch}}{\gamma(1 + k^2) + \Delta\beta_{ch}} \right]. \quad (16)$$

In the above relations we use the conditions $S_0^{(1)} = 0$ and $S_l^{(1)} = 0$ for $|l| > 1$. To the next order ($n = 2$, $l = 2$), we obtain the following second-harmonic modes (generated due to the self-interactions among modes):

$$[\phi_2^{(2)}, n_2^{(2)}, u_2^{(2)}, q_2^{(2)}]^T = [A, B, C, -(\beta_{ch}A + \beta_2)]^T \phi_1^{(1)2}. \quad (17)$$

The expressions for A , β_2 , B , and C are given in Appendix A [see Eqs. (A4)–(A7)]. The zeroth-harmonic modes that appear due to self-interaction among the modulated carrier wave are determined from the $l = 0$ component of the third-order ($n = 3$) relations and are found to be given by

$$\begin{aligned} [\phi_0^{(2)}, n_0^{(2)}, u_0^{(2)}, q_0^{(2)}]^T &= \left[D, \frac{1}{v_g} \left(E + \frac{2k^3}{\alpha_d^2 \omega^3} \right), E, \right. \\ &\quad \left. (-\beta_{ch}D + 2\beta_2) \right]^T |\phi_1^{(1)}|^2. \end{aligned} \quad (18)$$

The expressions of D and E are given in Appendix A [see Eqs. (A8) and (A9)]. Finally, we substitute the above derived relations into the third-order ($n = 3$) relations and obtain the following nonlinear Schrödinger equation (NLSE) for $\phi_1^{(1)} [= \psi]$:

$$i \frac{\partial \psi}{\partial \tau} + P \frac{\partial^2 \psi}{\partial \xi^2} + Q |\psi|^2 \psi + i \Gamma_{ch} \psi = 0, \quad (19)$$

with a dissipative term Γ_{ch} . In this NLSE, the group dispersion coefficient P [for the expression, see Eq. (A11)] is related to the curvature of the dispersion relation $\omega(k)$ [Eq. (14)], which is always negative for all wave numbers k . The coefficient of the nonlinear term Q [for the expression, see Eq. (A12)] is related to the nonlinear frequency shift. The dissipative term

$$\Gamma_{ch} = \frac{\beta_{ch} \Delta^2 k^2}{2\alpha_d[\gamma(1 + k^2) + \Delta\beta_{ch}]^2} \left(\frac{\omega_{pd}}{v_{ch}} \right) \quad (20)$$

is related to the dust charge variations. This expression shows that the dissipative term Γ_{ch} disappears for $\omega_{pd}/v_{ch} \approx 0$, as it should because this implies that the left-hand side of the charging equation (5) equals zero, i.e., $I_e + I_+ + I_- = 0$. In this case the dust charge Q_d is in the local equilibrium state, providing an approximate description of the charge state of a dust grain, and produces no dissipative effect (adiabatic dust charge variations). Also note that $\Gamma_{ch} = 0$ for $\Delta (= 1 - \delta_- - \delta_+ = z_d n_{d0}/n_{+0}) = 0$; this is expected as $\Delta = 0$ implies the absence of charged dust grains and consequently no charge variations. Hence, the dissipative term Γ_{ch} arises only due to the nonsteady (nonadiabatic) dust charge variations.

IV. EFFECTS OF DUST CHARGE VARIATIONS ON MODULATIONAL INSTABILITY

At very low frequencies, the DAW is nearly at zero frequency and Eq. (19) can be analyzed for its stability. In the above damped NLSE [Eq. (19)], the term $-Q|\psi|^2$ plays the role of a potential energy. The local maxima of this potential energy acts as an effective potential well in the plasma. The physics of this phenomena can be explained in the following manner. The high-frequency DAW reflects from the regions of high density, and the amplitude-dependent ponderomotive force forms a cavity (low-density regions) in the plasma. The DAW becomes trapped within these density-depleted regions (cavity) made by the wave itself. If a wave of amplitude ψ is trapped in a well, its energy $|\psi|^2$ will concentrate at the bottom of the well, but the energy concentration makes the well deeper by making this energy even larger. Thus the formation of the density depletion is an unstable physical process that occurs due to the energy localization in the medium. This process leads to an instability known as modulational instability (or caviton instability in the case of one dimension) [33,34]. Here, we briefly discuss this instability for DAW in electronegative dusty plasma and the effects of charge variations on this instability.

Let us assume that, in the presence of nonadiabatic dust-charge-variation-induced dissipation, the amplitude evolution equation (19) possesses the following plane-wave solution:

$$\psi = \psi_0(\tau) \exp \left[-i \int_0^\tau \delta(\tau') d\tau' \right], \quad (21)$$

where $\psi_0(\tau)$ and $\delta(\tau)$ are the amplitude of the pump carrier wave and the nonlinear frequency shift in the presence of dissipation. Substitution of this plane-wave solution in Eq. (19) yields the following two equations:

$$\frac{d\psi_0}{d\tau} + \Gamma_{ch} \psi_0 = 0 \Rightarrow \psi_0(\tau) = \psi_{00} \exp(-\Gamma_{ch} \tau), \quad (22)$$

and

$$\delta(\tau) = -Q|\psi_0(\tau)|^2 = -Q|\psi_{00}|^2 \exp(-2\Gamma_{\text{ch}}\tau), \quad (23)$$

where ψ_{00} is a real constant. Also note that $\psi_0(\tau) \rightarrow 0$ as $\tau \rightarrow \infty$, which implies that ψ_0 is stable. For stability analysis, we consider the perturbation about this stable solution in the following standard procedure:

$$\psi = [\psi_0(\tau) + \tilde{\psi}(\xi, \tau)] \exp \left[-i \int_0^\tau \delta(\tau') d\tau' \right], \quad (24)$$

where $\tilde{\psi}(|\tilde{\psi}| \ll \psi_0)$ is the perturbed amplitude of the modulated wave.

After the substitution of Eq. (24) in Eq. (19), one can show that the perturbation satisfies the following linearized equation:

$$i \frac{\partial \tilde{\psi}}{\partial \tau} + P \frac{\partial^2 \tilde{\psi}}{\partial \xi^2} + Q|\psi_0|^2(\tilde{\psi} + \tilde{\psi}^*) + i\Gamma_{\text{ch}}\tilde{\psi} = 0, \quad (25)$$

where $\tilde{\psi}^*$ is complex conjugate of $\tilde{\psi}$. Let us put $\tilde{\psi} = \tilde{\psi}_R + i\tilde{\psi}_I$, and then the above equation satisfies the following two coupled equations:

$$\frac{\partial \tilde{\psi}_I}{\partial \tau} = P \frac{\partial^2 \tilde{\psi}_R}{\partial \xi^2} + 2Q|\psi_0|^2\tilde{\psi}_R - \Gamma_{\text{ch}}\tilde{\psi}_I, \quad (26)$$

and

$$\frac{\partial \tilde{\psi}_R}{\partial \tau} = -P \frac{\partial^2 \tilde{\psi}_I}{\partial \xi^2} - \Gamma_{\text{ch}}\tilde{\psi}_R. \quad (27)$$

Finally, the space-time dependence of the perturbation of the form $\tilde{\psi} \sim \exp(i\vartheta)$ [where $\vartheta = \tilde{k}\xi - \int_0^\tau \tilde{\omega}(\tau') d\tau'$ is the modulated phase with $\tilde{k}(\ll k)$ and $\tilde{\omega}(\ll \omega)$ being the modulation frequency and wave number, respectively) yields the dispersion relation:

$$(\tilde{\omega} + i\Gamma_{\text{ch}})^2 = P^2\tilde{k}^4 - 2PQ|\psi_0|^2\tilde{k}^2.$$

This dispersion relation shows that the nonsteady (nonadiabatic) dust-charge-variation-induced dissipation provides the usual damping of the wave. The system is stable for $PQ < 0$. However, there is a possibility of instability if $PQ > 0$: both P and Q are of the same sign, here $P < 0$ for all $\tilde{k}$ [Eq. (A11)], so to make $PQ > 0$ we must have $Q < 0$. In this case of nonadiabatic dust charge variations, the instability could occur if

$$\tilde{k}^2 < \tilde{k}_{cr}^2 = \left(\frac{2Q}{P} \right) |\psi_0|^2 = \left(\frac{2Q}{P} \right) |\psi_{00}|^2 \exp(-2\Gamma_{\text{ch}}\tau),$$

provided with these values of $\tilde{k}$ the inequality

$$\Gamma_{\text{ch}} < \sqrt{P^2\tilde{k}^2 \left(\frac{\tilde{k}_{cr}^2}{\tilde{k}^2} - 1 \right)}$$

must hold. This determines the maximum time,

$$\tau_{\text{max}} = \frac{1}{2\Gamma_{\text{ch}}} \ln \left(\frac{2PQ\tilde{k}^2|\psi_{00}|^2}{\Gamma_{\text{ch}}^2 + P^2\tilde{k}^4} \right), \quad (28)$$

to observe instability. Thus the instability growth will cease for $\tau \geq \tau_{\text{max}}$. For $PQ > 0$, by setting $\tilde{\omega} = i\Gamma$, we obtain the dispersion relation,

$$(\Gamma + \Gamma_{\text{ch}})^2 = 2PQ|\psi_0|^2\tilde{k}^2 - P^2\tilde{k}^4,$$

for the instability growth rate. The maximum growth rate is obtained by taking the derivative of the above dispersion relation with respect to $\tilde{k}^2$, and setting this to zero. This yields

$$\tilde{k}_{\text{max}}^2 = \left(\frac{Q}{P} \right) |\psi_0|^2 = \left(\frac{Q}{P} \right) |\psi_{00}|^2 \exp(-2\Gamma_{\text{ch}}\tau),$$

and, with these values of $\tilde{k}_{\text{max}}^2$, the maximum growth rate becomes

$$\begin{aligned} \Gamma_{\text{max}}(\text{nonadiabatic}) &= \sqrt{2}|Q||\psi_0|^2 - \Gamma_{\text{ch}} \\ &= \sqrt{2}|Q||\psi_{00}|^2 \exp(-2\Gamma_{\text{ch}}\tau) - \Gamma_{\text{ch}}. \end{aligned} \quad (29)$$

The configuration is unstable when $\sqrt{2}|Q||\psi_0|^2 > \Gamma_{\text{ch}}$.

In the case of adiabatic charge variations, $\omega_{\text{pd}}/\nu_{\text{ch}} = 0$, i.e., $\Gamma_{\text{ch}} = 0$, and there is no dissipation in the system. In this case, one can easily find the dispersion relation,

$$\tilde{\omega}^2 = P^2\tilde{k}^4 - 2PQ|\psi_{00}|^2\tilde{k}^2,$$

of the modulated wave. The instability criteria for $PQ > 0$ of the modulated wave then becomes $\tilde{k}^2 < \tilde{k}_{cr}^2 = (2Q/P)|\psi_{00}|^2$. In this case,

$$\begin{aligned} \Gamma_{\text{max}}(\text{adiabatic}) &= \lim_{\Gamma_{\text{ch}} \rightarrow 0} \Gamma_{\text{max}}(\text{nonadiabatic}) \\ &= \sqrt{2}|Q||\psi_{00}|^2 \end{aligned} \quad (30)$$

is the maximum growth rate. Comparing Eq. (30) with that of the nonadiabatic case [Eq. (29)], we obtain

$$\Gamma_{\text{max}}(\text{adiabatic}) > \Gamma_{\text{max}}(\text{nonadiabatic}),$$

as $\Gamma_{\text{ch}} > 0$.

All the above investigations reveal that the nonadiabatic dust-charge-variation-induced dissipation reduces the instability growth and therefore introduces a characteristic time scale to make the instability occur. Thus the nonlinear DAW becomes modulationally unstable within the time interval $[0, \tau_{\text{max}}]$ for $PQ > 0$. The nonlinear saturation of such DAWs can give rise to bright (envelope) soliton structures. Therefore, the value of τ_{max} provides an estimation of a characteristic time to observe a bright soliton in a dusty plasma with nonadiabatic dust charge variations. However, in the adiabatic limit ($\Gamma_{\text{ch}} \rightarrow 0$) $\tau_{\text{max}} \rightarrow \infty$. Thus, in the case of steady-state (adiabatic) charge variations, there does not exist any characteristic time scale to observe the bright soliton. In the stable region $PQ < 0$, it is well known that a finite amplitude DAW can not form an envelope soliton [35]; instead, it forms a dark soliton structure.

We now investigate the group dispersion coefficient P and nonlinear coefficient Q and also the instability conditions numerically for wave number k . The computations are carried out for the representative parameters of the $Ar^+ - F^-$ electronegative dusty plasmas, used for ultrafine and highly selective etching of polysilicons [2,10,15]. The parameters used in the computations are summarized as follows: $n_{+0} \sim 4 \times 10^{17} \text{ m}^{-3}$, $n_{d0} \sim 4 \times 10^{13} \text{ m}^{-3}$, $T_e = 2 \text{ eV}$, $T_- = 0.1 \text{ eV}$, $T_+ = 0.2 \text{ eV}$, $m_- \sim 3.2 \times 10^{-26} \text{ kg}$, $m_+ \sim 6.69 \times 10^{-26} \text{ kg}$, and the dust mass density $\rho_d \sim 1.5 \times 10^3 \text{ kg m}^{-3}$. The dusty plasma parameter z is computed for two different dust grain radii r_d and four cases of negative ion content $\delta_- (= n_{-0}/n_{+0})$. The cases are [15] (a) $r_d = 0.1 \mu\text{m}$, $\delta_- = 0.1$;

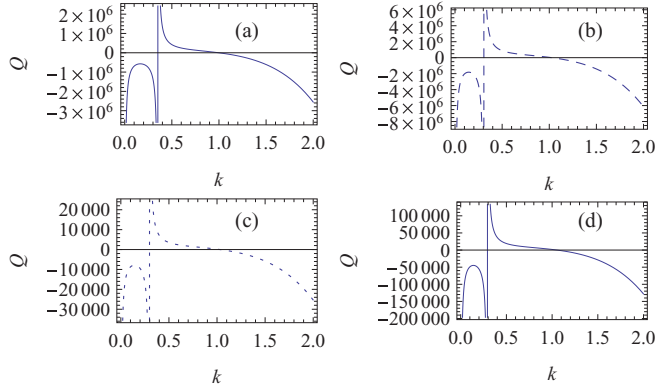


FIG. 1. (Color online) Variations of Q with wave number k for different plasma parameters. The plasma parameters for different figures are (a) $r_d = 0.1 \mu\text{m}$, $\delta_- = 0.1$, $z = 3.3$, and $\delta_+ = 0.854$; (b) $r_d = 0.1 \mu\text{m}$, $\delta_- = 0.4$, $z = 2.98$, and $\delta_+ = 0.56$; (c) $r_d = 1 \mu\text{m}$, $\delta_- = 0.1$, $z = 2.89$, and $\delta_+ = 0.498$; and (d) $r_d = 1 \mu\text{m}$, $\delta_- = 0.4$, $z = 2.42$, and $\delta_+ = 0.337$.

(b) $r_d = 0.1 \mu\text{m}$, $\delta_- = 0.4$; (c) $r_d = 1 \mu\text{m}$, $\delta_- = 0.1$; and (d) $r_d = 1 \mu\text{m}$, $\delta_- = 0.4$, respectively.

Different curves (a), (b), (c), and (d) in Fig. 1 show that the magnitude of the coefficient of nonlinearity Q increases with the increase of negative ion content. However, a comparative study between the different curves shows that Q decreases for larger dust grains. The magnitude of the coefficient of group dispersion P increases with the increase of the proportion of the negative ions for larger dust grains. But comparative study shows that the difference is insignificant for smaller dust grains. These are shown graphically in Fig. 2.

The nonlinear DAW becomes modulationally unstable only if $Q < 0$ (as P is always negative). Figure 1 reveals that $Q < 0$ for the wave number $k < 0.5$ and also for $k > 1$. Thus DAW becomes modulationally unstable in the very-small-wavelength perturbation and also in the sufficiently-long-wavelength perturbation. The nonsteady dust-charge-variation-induced dissipation Γ_{ch} decreases with the increase of the negative ion concentration, as shown in Fig. 3. This figure also shows that the dissipation is very low for larger dust grains.

V. SOLITON SOLUTIONS

In the above damped NLSE [Eq. (19)], the group dispersion coefficient P [Eq. (A11)] is always negative, but the nonlinear coefficient Q [Eq. (A12)] can be both positive and negative depending on the wave number k . Thus, to find the soliton solutions of the damped NLSE [Eq. (19)], we recast the equation in the following normal form:

$$i \frac{\partial \psi}{\partial \bar{\tau}} - \frac{1}{2} \frac{\partial^2 \psi}{\partial \bar{\xi}^2} + \kappa |\psi|^2 \psi + i \varepsilon(\psi) = 0, \quad (31)$$

where $\bar{\tau} = |Q|\tau$, $\bar{\xi} = \xi \sqrt{|Q|/2|P|}$ and $\varepsilon(\psi) = \Gamma_{\text{ch}}\psi/|Q|$. This equation exhibits a bright or envelope soliton for $\kappa = -1$, whereas it exhibits a dark soliton for $\kappa = +1$. Equation (31) is solved numerically for $\kappa = \pm 1$, and the solutions are shown graphically in Figs. 4 and 5. However, in the proceeding subsections, we have solved this equation analytically.

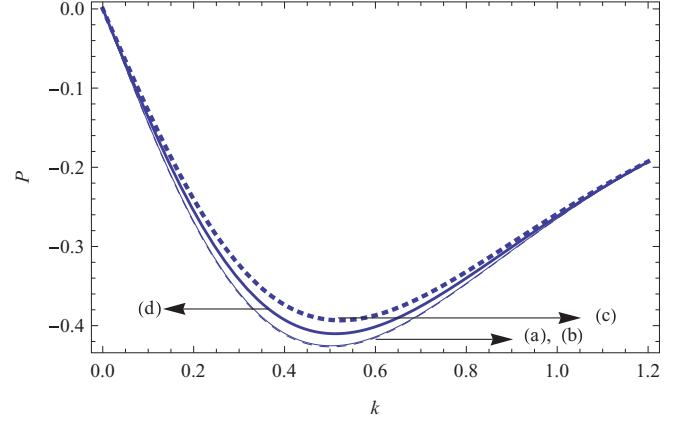


FIG. 2. (Color online) Variations of group dispersion coefficient P with wave number k for different plasma parameters. The plasma parameters for the different figures are the same as in Fig. 1.

A. Bright soliton: adiabatic charge variation

In the case of adiabatic dust charge variations, $\omega_{\text{pd}}/\nu_{\text{ch}} \approx 0$, and then, in Eq. (31), we can put $\varepsilon(\psi) = 0$. Thus, in the adiabatic limit, we have the usual NLSE with $\kappa = -1$. Let us assume a solution of the form $\psi(\bar{\xi}, \bar{\tau}) = \rho(\bar{\xi}, \bar{\tau}) \exp[i\varphi(\bar{\xi}, \bar{\tau})]$ and then solve the ordinary differential equations for φ and ρ subject to the boundary condition $\rho \rightarrow 0$ as $\bar{\xi} \rightarrow \pm\infty$. We finally obtain the following single envelope (bright) soliton, which in terms of actual parameters reads as

$$\psi(\xi, \tau) = a \cosh^{-1} \left[a \sqrt{\frac{|Q|}{2|P|}} (\xi + \sqrt{2|PQ|\chi}\tau) \right] \times \exp \left\{ i \sqrt{\frac{|Q|}{2|P|}} \left[\chi \xi - \sqrt{\frac{|PQ|}{2}} (a^2 - \chi^2) \tau \right] \right\}, \quad (32)$$

where a and χ are two soliton parameters in which a is the amplitude of the soliton. This shows that the disturbances resemble the soliton shape with an exponential factor making oscillation between a maxima and a minima. The resultant structure is the envelope excitation of DAW. The graphical

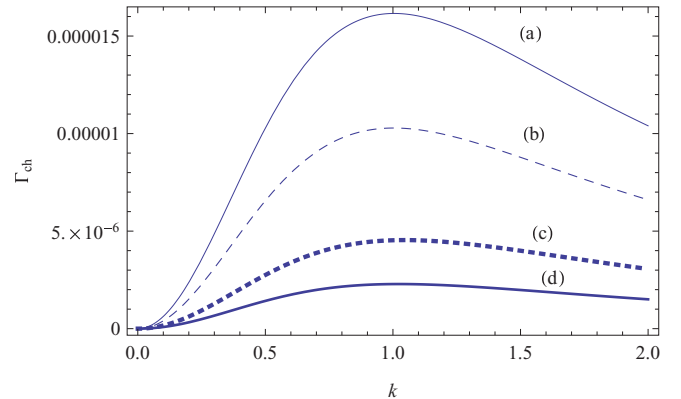


FIG. 3. (Color online) Variations of nonsteady dust-charge-variation-induced dissipation Γ_{ch} with wave number k for different plasma parameters. The plasma parameters for the different figures are the same as in Fig. 1.

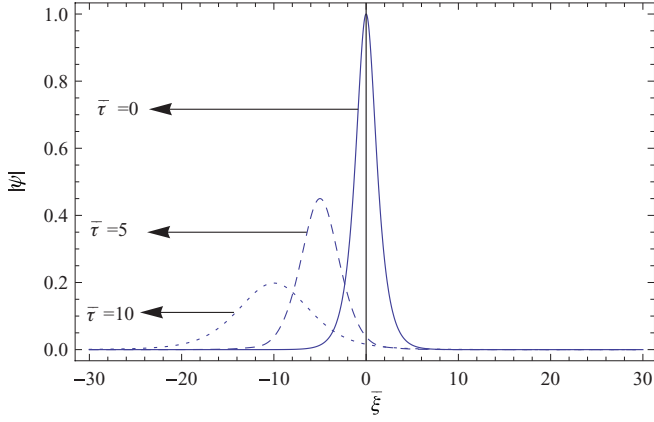


FIG. 4. (Color online) Bright soliton in the presence of dissipation due to nonsteady dust charge variation. Numerical solution of Eq. (31) for $\kappa = -1$, where $\bar{\tau} = |Q|\tau$ and $\bar{\xi} = \xi\sqrt{|Q|/2|P|}$.

representation of this single bright soliton is shown in Fig. 6 for different plasma parameters.

B. Bright soliton: nonadiabatic charge variation

In the case of nonadiabatic dust charge variations, $\omega_{pd}/\nu_{ch} \neq 0$, and if this dissipation is weak [here it is indeed weak, as $\omega_{pd}/\nu_{ch} \sim O(\epsilon^2)$], we can solve the above damped NLSE [Eq. (19)] perturbatively by taking $\varepsilon(\psi)$ as a small perturbation. Therefore, to solve it, let us take the general solution of the perturbed soliton in the following form [36,37]:

$$\psi(\bar{\xi}, \bar{\tau}) = a(\bar{\tau}) \cosh^{-1} \{a(\bar{\tau})[\bar{\xi} + b(\bar{\tau})]\} \exp[i\bar{\xi}\bar{\chi}(\bar{\tau}) - i\sigma(\bar{\tau})], \quad (33)$$

where a , b , σ , and $\bar{\chi}$ are the soliton parameters. Finally, applying the conservation laws for the NLSE (conserved integral relations [36,37]), we obtain the following bright soliton (envelope soliton) in the presence of nonsteady dust

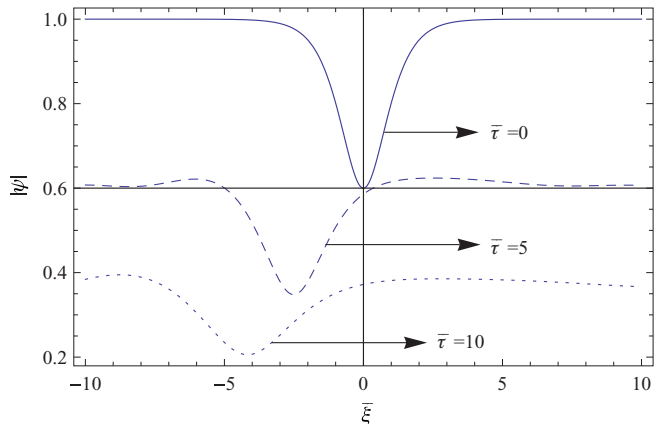


FIG. 5. (Color online) Dark soliton in the presence of dissipation due to nonsteady dust charge variation. Numerical solution of Eq. (31) for $\kappa = +1$. The independent variables are the same as in Fig. 4.

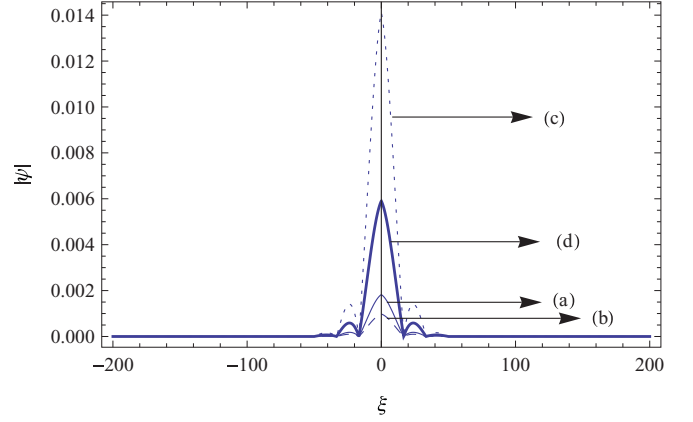


FIG. 6. (Color online) Bright (envelope) soliton. Numerical solution of Eq. (19) with $\Gamma_{ch} = 0$ for $PQ > 0$ with $k = 0.2$. The plasma parameters for the different figures are the same as in Fig. 1.

charge variations, which in terms of actual variable reads as

$$\begin{aligned} \psi(\xi, \tau) = & a_0 \exp(-2\Gamma_{ch}\tau) \cosh^{-1} \\ & \times \left[a_0 \exp(-2\Gamma_{ch}\tau) \sqrt{\frac{|Q|}{2|P|}} (\xi + \sqrt{2|PQ|}\chi_0\tau) \right] \\ & \times \exp \left[i \sqrt{\frac{|Q|}{2|P|}} \left(\chi_0\xi + \sqrt{\frac{|PQ|}{2}} \chi_0^2\tau \right. \right. \\ & \left. \left. + a_0^2 \left[\frac{\exp(-4\Gamma_{ch}\tau) - 1}{4\Gamma_{ch}} \right] \right) \right], \end{aligned} \quad (34)$$

where a_0 and χ_0 are the initial values of a and χ , respectively. In the limit $\Gamma_{ch} \rightarrow 0$, we recover the previous result [Eq. (32)] (with $a_0 = a$ and $\chi_0 = \chi$). It is clear that as time elapses the amplitude of the bright (envelope) soliton decreases exponentially with decay rate $\sim 2\Gamma_{ch}$. The numerical solution in Fig. 4 also shows a similar nature. Thus the nonadiabatic dust charge variation has a damping effect on the bright soliton structure in a dusty plasma.

C. Dark soliton: adiabatic charge variation

As before, in this case we have the usual NLSE with $\kappa = +1$. To obtain the dark soliton solution, proceeding as before, subject to the boundary conditions $\rho \rightarrow u_0$ (a constant), we obtain the following single dark soliton [36,37,39]:

$$\psi(\xi, \tau) = u_0[\eta \tanh(\zeta) + i\sqrt{1-\eta^2}] \exp(i|Q|u_0^2\tau), \quad (35)$$

where $\zeta = u_0\eta(\sqrt{|Q|/2|P|}\xi + u_0\tau|Q|\sqrt{1-\eta^2})$. The parameters u_0 and η represent the amplitude and depth of the dip of the dark soliton. The single dark soliton is shown graphically in Fig. 7 for different plasma parameters. Dark solitons are called *gray* solitons when $|\eta| < 1$. On the other hand, the $|\eta| = 1$ case corresponds to the *black* solitons or stationary shock solution given by

$$\psi(\xi, \tau) = u_0 \tanh[u_0\sqrt{|Q|/2|P|}\xi] \exp(i|Q|u_0^2\tau). \quad (36)$$

In this case an input pulse exhibits an intensity “hole” at the center, and thus, sometimes, it is called an envelope hole.

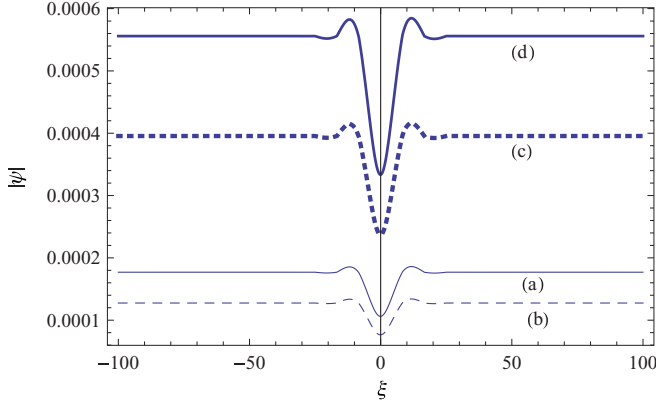


FIG. 7. (Color online) Dark soliton. Numerical solution of Eq. (19) with $\Gamma_{ch} = 0$ for $PQ < 0$ with $k = 0.5$. The plasma parameters for the different figures are the same as in Fig. 1.

D. Dark soliton: nonadiabatic charge variation

In the presence of weak dissipation $\varepsilon(\psi)$, we assume $\psi(\xi, \tau) = \psi[u_0(\tau), \eta(\tau); \xi, \tau]$ using the above solution (36) and then applying the conservation laws for the NLSE (conserved integral relations [38,39]), we obtain the following dark soliton:

$$\psi(\xi, \tau) = u_0(\tau) \{ \eta(\tau) \tanh[\zeta(\tau)] + i\sqrt{1 - \eta(\tau)^2} \} \times \exp(i|Q|u_0(\tau)^2\tau), \quad (37)$$

in the the presence of nonsteady dust charge variations. In the above $\zeta(\tau) = u_0(\tau)\eta(\tau)(\sqrt{|Q|/2|P|}\xi + u_0(\tau)\tau|Q|\sqrt{1 - \eta(\tau)^2})$ and

$$u_0(\tau) = u(0) \exp(-\Gamma_{ch}\tau), \quad \eta(\tau) = \eta(0) \exp(-\Gamma_{ch}\tau), \quad (38)$$

where $u(0)$ and $\eta(0)$ are the initial values of u_0 and η , respectively. The dark soliton amplitude also decays exponentially with time with decay rate $\sim \Gamma_{ch}$ due to the nonsteady dust charge variations. The numerical solution in Fig. 5 also shows a similar nature. In this case the decay rate is half of that of the bright soliton. In the limit $\Gamma_{ch} \rightarrow 0$, we recover the usual single dark soliton [Eq. (35)].

VI. DISCUSSION

In the preceding sections, we see that the dynamics of the modulated DAW in charge varying electronegative dusty plasma is governed by a damped NLSE. The damping term appears due to the nonsteady (nonadiabatic) dust charge variations. To study the effects of nonsteady dust charge variations on DAW modulations, we have assumed that $\omega_{pd}/\nu_{ch} \sim O(\epsilon^2)$. This assumption is valid if the dust oscillation period is comparable with the dust charging frequency, i.e., the conditions $\omega_{pd}/\nu_{ch} \ll 1$, $\neq 0$ are met. The specified $Ar^+ - F^-$ laboratory plasma parameters yield $\omega_{pd}/\nu_{ch} \sim 1.4 \times 10^{-2}$ for smaller dust grains ($r_d = 0.1 \mu\text{m}$). Thus, the condition for nonadiabaticity is indeed satisfied. However, for larger ($r_d > 1 \mu\text{m}$) dust grains the ratio becomes $< 4 \times 10^{-4}$ and the conditions could be violated. In this case, one can consider the adiabatic dust charge variations.

The present investigation shows that the DAWs are trapped in a hole created by the wave itself in electronegative dusty

plasma and they become modulationally unstable in the region $PQ > 0$. In this unstable region DAWs admit a bright (envelope) soliton, as shown in Fig. 6. In the stable region $PQ < 0$, DAWs admit a dark soliton, as shown in Fig. 7. The presence of negative ions decreases the magnitude of the bright soliton amplitude (depth of the dip of the dark soliton), and, as a consequence, the width of both solitons increases. Also, with the increase of the dust grain radius, the magnitude of the soliton amplitude (depth of the dip) increases, whereas width increases [see the comparative study between the solitons (a) and (b) and the solitons (c) and (d) in Figs. 6 and 7].

In the case of steady-state (adiabatic) dust charge variations, the dust charge instantaneously reaches its equilibrium value and thus does not produce any dissipative effect. However, in the case of nonsteady dust charge variations the growth rate of modulational instability is reduced. Actually, due to the delayed charging (nonadiabatic dust charge variations), i.e., the nonzero value of ω_{pd}/ν_{ch} , there is an extra contribution from the dust charge Q_d through the charging equation (5). This causes a decay of the dust charge magnitude and, consequently, plays the role of the dissipative mechanism on the dust fluid, causing dust motion by decreasing the driving force ($\propto Q_d$). This dissipation produces the damping effects on collective phenomena in a dusty plasma. The damping effect opposes the energy localization in the medium (that causes the instability), and as a result the instability growth rate is reduced.

The nonsteady dust charge variations also introduce a characteristic time $\tau_{\max}$ to observe modulational instability, i.e., to observe envelope solitons, which cannot be found in dusty plasma with a constant dust charge [29], and also for steady-state charge variations. This is a unique feature of a truly dusty plasma. In the presence of dissipation, the analytical solutions show that the nonlinear DAW can propagate in the form of bright and dark solitons with decaying amplitude (and amplifying width). The decay rate of the bright soliton is twice the decay rate of the dark soliton. The numerical solutions also reveal the same nature as that shown in Figs. 4 and 5 and thus confirm the analytical results.

It should be noted that the physical parameters for the adiabatic assumption are different from those of the nonadiabatic assumption. As a result, both bright (envelope) and dark solitons for the adiabatic case are quantitatively different from those of the nonadiabatic case.

VII. CONCLUSION

On the basis of dust fluid equations, we have analyzed the effects of dust charge variations and negative ions on DAW modulations in a dusty plasma. Our investigation shows that there is a possibility of the trapping of DAWs in a hole created by the wave itself in the medium. The parameter region, where DAWs become modulationally unstable, is specified. In the presence of nonsteady (nonadiabatic) dust charge variations, the dynamics of the modulated wave is governed by a damped NLSE. The modulated wave exhibits both dissipative (here damped) bright (envelope) and dark solitons. Negative ion content reduces (increases) the soliton amplitude (width). It should be noted that the balance between dispersion and nonlinearity gives rise to a similar type of bright (dark) solitons described by the NLSE in optical fiber communication

systems, provided that the pulse propagates in the anomalous group-velocity dispersion regime. The dissipative bright (dark) solitons are also observed in optical fiber, where the dissipation occurs due to the fiber losses in the system [36–39]. Here, the decay rate of the dark soliton is half of that of the bright soliton, which is similar to the observed dissipative optical soliton in the communication system [36–39].

Finally, we stress that the results of our investigation could be useful in understanding the features of modulated DAW packets in a dissipative electronegative dusty plasma. Recently, a dissipative dark soliton was observed in a strongly coupled complex plasma [40,41]. In that experiment, the dissipation (damping) occurs due to the neutral drag, whereas here the damping is due to nonadiabatic charge variations. In this connection, we must mention that we have not encountered any experimental observations of bright (envelope) and dark solitons in a dissipative electronegative dusty plasma. It would be very interesting to look at this situation in a laboratory. We hope that, in the future, such a type of dissipative soliton could be observed in electronegative dusty plasma experiments.

APPENDIX A: EXPRESSIONS OF DIFFERENT COEFFICIENTS

$$\begin{aligned} A_+ &= \frac{\delta_+ \sigma_+ \exp(-z)}{(z + \sigma_+)} \sqrt{\frac{m_+ T_e}{m_e T_+}}, \\ A_- &= \frac{\delta_- \exp(-z/\sigma_-)}{(z + \sigma_+)} \sqrt{\frac{\sigma_+ \sigma_- m_+}{m_-}}, \end{aligned} \quad (A1)$$

$$\beta_{ch} = \frac{(z + \sigma_+)(1 + \sigma_+ + \mu_2^-)}{z\sigma_+(1 + z + \sigma_+ + \mu_1^-)}, \quad (A2)$$

$$\mu_1^- = \frac{(z + \sigma_+)(1 - \sigma_-)A_-}{\sigma_-}, \quad (A3)$$

$$\mu_2^- = \frac{\sigma_+(1 - \sigma_-)A_-}{\sigma_-}.$$

Note that the equilibrium current balance equation is given by $I_{e0} + I_{+0} + I_{-0} = 0 \Rightarrow A_+ + A_- = 1$:

$$A = \frac{1}{3\gamma k^2} \left[\Delta\beta_2 - \frac{3\gamma k^2(1 + k^2)}{2\alpha_d \omega^2} - \frac{1}{2} \left(\frac{\delta_-}{\sigma_-^2} + \delta_+ - \frac{1}{\sigma_+^2} \right) \right], \quad (A4)$$

$$\begin{aligned} \beta_2 &= \frac{\beta_{ch}}{2(A_+ + \frac{A_-}{\sigma_-} + \frac{1}{\sigma_+})} \left[\frac{1}{\sigma_+^2} - (1 - z\beta_{ch})^2 \left(A_+ + \frac{A_-}{\sigma_-^2} \right) \right. \\ &\quad \left. - \frac{2z\beta_{ch}}{\sigma_+(z + \sigma_+)} \right], \end{aligned} \quad (A5)$$

$$B = \frac{k^2}{2\alpha_d \omega^2} \left(\frac{3k^2}{\alpha_d \omega^2} - \beta_{ch} - 2A \right), \quad (A6)$$

$$C = \frac{k[\gamma(1 + k^2) - 2\Delta A]}{2\alpha_d \Delta \omega}, \quad (A7)$$

$$D = \frac{2}{\gamma + \Delta(\beta_{ch} - \frac{1}{\alpha_d v_g^2})} \left\{ \Delta\beta_2 + \frac{\Delta\beta_{ch} k^2}{\alpha_d \omega^2} - \frac{[\gamma(3 + k^2) + 2\Delta\beta_{ch}]}{2\alpha_d v_g^2} + \frac{1}{2} \left(\frac{1}{\sigma_+^2} + \delta_+ - \frac{\delta_-}{\sigma_-^2} \right) \right\}, \quad (A8)$$

$$E = \frac{[\gamma(1 + k^2) - \Delta D]}{\alpha_d \Delta v_g}, \quad (A9)$$

$$\begin{aligned} \beta_3 &= \frac{\beta_{ch}}{(A_+ + \frac{A_-}{\sigma_-} + \frac{1}{\sigma_+})} \left\{ (A + D) \left\{ \left[\frac{1}{\sigma_+^2} - \frac{2z\beta_{ch}}{\sigma_+(z + \sigma_+)} \right] - \left(A_+ + \frac{A_-}{\sigma_-^2} \right) (1 - z\beta_{ch})^2 \right\} \right. \\ &\quad + 3z\beta_2 \left(A_+ + \frac{A_-}{\sigma_-^2} \right) (z\beta_{ch} - 1) + \frac{3z}{\sigma_+(z + \sigma_+)} \left(\beta_2 + \frac{\beta_{ch}}{2\sigma_+} \right) - \frac{1}{2\sigma_+^3} \\ &\quad \left. - \frac{1}{2} \left(A_+ + \frac{A_-}{\sigma_-^3} \right) (1 - z\beta_{ch})^3 \right\}. \end{aligned} \quad (A10)$$

The group dispersion coefficient P is given by

$$P = - \left(\frac{3\gamma \alpha_d \omega^3}{2\Delta k^2} \right) \left[\frac{\gamma + \Delta\beta_{ch}}{\gamma(1 + k^2) + \Delta\beta_{ch}} \right] = \frac{1}{2} \frac{d^2 \omega(k)}{dk^2}. \quad (A11)$$

The coefficient of the nonlinear term Q is given by

$$\begin{aligned} Q &= - \frac{\alpha_d \omega^3}{2\Delta k^2} \left\{ \Delta\beta_3 - (A + D) \left(\frac{\delta_-}{\sigma_-^2} + \delta_+ - \frac{1}{\sigma_+^2} \right) + \gamma(1 + k^2) \left(B + \frac{E}{v_g} + \frac{2k^3}{v_g \alpha_d^2 \omega^3} \right) \right. \\ &\quad \left. - \frac{1}{2} \left(\frac{\delta_-}{\sigma_-^3} + \delta_+ + \frac{1}{\sigma_+^3} \right) + \frac{2\Delta k^2}{\alpha_d \omega^2} \left[\frac{k(C + E)}{\omega} + \beta_{ch}(A + D) - 2\beta_2 \right] \right\}. \end{aligned} \quad (A12)$$

APPENDIX B: OUTLINES OF CALCULATIONS

Substitution of Eqs. (7)–(9) in Eqs. (2)–(5) yields the following n th-order reduced set of equations:

$$-i\omega n_l^{(n)} + ikl u_l^{(n)} + \frac{\partial}{\partial \xi} (-v_g n_l^{(n-1)} + u_l^{(n-1)}) + \frac{\partial n_l^{(n-2)}}{\partial \tau} + \sum_{n'=1}^{\infty} \sum_{l'=-\infty}^{\infty} \left\{ \frac{\partial}{\partial \xi} [n_{l-l'}^{(n-n'-1)} u_{l'}^{(n')}] + ikl n_{l-l'}^{(n-n')} u_{l'}^{(n')} \right\} = 0, \quad (\text{B1})$$

$$\begin{aligned} & -i\omega n_l^{(n)} + \frac{ikl}{\alpha_d} \phi_l^{(n)} + v_g \frac{\partial u_l^{(n-1)}}{\partial \xi} + \frac{1}{\alpha_d} \frac{\partial \phi_l^{(n-1)}}{\partial \xi} - \frac{\partial u_l^{(n-2)}}{\partial \tau} \\ & = \sum_{n'=1}^{\infty} \sum_{l'=-\infty}^{\infty} \left[n_{l-l'}^{(n-n'-1)} \frac{\partial u_{l'}^{(n')}}{\partial \xi} + \frac{1}{\alpha_d} q_{l-l'}^{(n-n'-1)} \frac{\partial \phi_{l'}^{(n')}}{\partial \xi} + ikl' u_{l-l'}^{(n-n')} u_{l'}^{(n')} + \frac{ikl'}{\alpha_d} q_{l-l'}^{(n-n')} \phi_{l'}^{(n')} \right], \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \gamma \left[\frac{\partial^2 \phi_l^{(n-2)}}{\partial \xi^2} + 2ikl \frac{\partial \phi_l^{(n-1)}}{\partial \xi} - k^2 l^2 \phi_l^{(n)} \right] & = \gamma \phi_l^{(n)} - \Delta q_l^{(n)} + \Delta n_l^{(n)} + \frac{1}{2} \sum_{n'=1}^{\infty} \sum_{l'=-\infty}^{\infty} \left[\left(\frac{\delta_-}{\sigma_-^2} + \delta_+ - \frac{1}{\sigma_+^2} \right) \phi_{l-l'}^{(n-n')} \phi_{l'}^{(n')} \right. \\ & \quad \left. - 2\Delta n_{l-l'}^{(n-n')} q_{l'}^{(n')} \right] + \frac{1}{6} \left(\frac{\delta_-}{\sigma_-^3} + \delta_+ + \frac{1}{\sigma_+^3} \right) \sum_{n'=1}^{\infty} \sum_{l'=-\infty}^{\infty} \sum_{n''=1}^{\infty} \sum_{l''=-\infty}^{\infty} \phi_{l-l'-l''}^{(n-n'-n'')} \phi_{l'}^{(n')} \phi_{l''}^{(n'')}, \end{aligned} \quad (\text{B3})$$

$$\begin{aligned} -i\omega l \left(\frac{\omega_{pd}}{\nu_{ch}} \right) \left(\frac{1 + \sigma_+ + \mu_2^-}{\sigma_+ \beta_{ch}} \right) q_l^{(n-2)} & = - \left(\frac{z}{z + \sigma_+} + zA_+ + \frac{zA_-}{\sigma_-} \right) q_l^{(n)} - \left(\frac{1}{\sigma_+} + A_+ + \frac{A_-}{\sigma_-} \right) \phi_l^{(n)} \\ & + \sum_{n'=1}^{\infty} \sum_{l'=-\infty}^{\infty} \frac{1}{2} \left[\left(\frac{1}{\sigma_+^2} - A_+ - \frac{A_-}{\sigma_-^2} \right) \phi_{l-l'}^{(n-n')} \phi_{l'}^{(n')} - \left(A_+ + \frac{A_-}{\sigma_-^2} \right) z^2 q_{l-l'}^{(n-n')} q_{l'}^{(n')} \right. \\ & + \frac{2z}{\sigma_+(z + \sigma_+)} q_{l-l'}^{(n-n')} \phi_{l'}^{(n')} - 2z \left(A_+ + \frac{A_-}{\sigma_-^2} \right) \phi_{l-l'}^{(n-n')} q_{l'}^{(n')} \left. \right] - \frac{1}{6} \sum_{n'=1}^{\infty} \sum_{l'=-\infty}^{\infty} \sum_{n''=1}^{\infty} \sum_{l''=-\infty}^{\infty} \\ & \left[\left(\frac{1}{\sigma_+^3} + A_+ + \frac{A_-}{\sigma_-^3} \right) \phi_{l-l'-l''}^{(n-n'-n'')} \phi_{l'}^{(n')} \phi_{l''}^{(n'')} + \left(A_+ + \frac{A_-}{\sigma_-^3} \right) z^3 q_{l-l'-l''}^{(n-n'-n'')} q_{l'}^{(n')} q_{l''}^{(n'')} \right. \\ & + \frac{3z}{\sigma_+^2(z + \sigma_+)} q_{l-l'-l''}^{(n-n'-n'')} \phi_{l'}^{(n')} \phi_{l''}^{(n'')} + 3z \left(A_+ + \frac{A_-}{\sigma_-^3} \right) \phi_{l-l'-l''}^{(n-n'-n'')} \phi_{l'}^{(n')} q_{l''}^{(n'')} \\ & \left. + 3z^2 \left(A_+ + \frac{A_-}{\sigma_-^3} \right) \phi_{l-l'-l''}^{(n-n'-n'')} q_{l'}^{(n')} q_{l''}^{(n'')} \right]. \end{aligned} \quad (\text{B4})$$

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